RELATIVISTIC NUCLEAR PHYSICS WITH THE SPECTATOR MODEL

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ABSTRACT

The spectator model, a general approach to the relativistic treatment of nuclear physics problems in which spectators to nuclear interactions are put on their mass-shell, will be defined and described. The approach grows out of the relativistic treatment of two and three body systems in which one particle is off-shell, and recent numerical results for the NN interaction will be presented. Two meson-exchange models, one with only 4 mesons $(\pi, \sigma, \rho, \omega)$ but with a 25% admixture of γ^5 coupling for the pion, and a second with 6 mesons $(\pi, \sigma, \rho, \omega, \delta, and\ \eta)$ but a pure $\gamma^5\gamma^\mu$ pion coupling, are shown to give very good quantitative fits to NN scattering phase shifts below 400 MeV, and also a good description of the $p\ ^{40}Ca$ elastic scattering observables.

INTRODUCTION

This talk will report on an approach to the relativistic treatment of nuclear systems which has grown out of work using relativistic equations with one particle off-shell. The essence of this approach is that the relativistic series of Feynman diagrams describing any nuclear process can always be reorganized so that only the particles which are interacting are off-shell, and all other particles, which are spectators to the interaction, can be put on-shell. For two and three nucleon systems this can always be done so that only one particle is off-shell, and the amplitudes required are either covariant vertex functions or covariant scattering amplitudes. With modifications, it

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appears that this general approach can be extended to many body systems, and I now refer to it as the spectator model.

Before turning to the details, a few words of philosophy are in order. It is assumed that there are at least two distance scales in nuclear physics. At large distances, from one to two fermis and beyond, it is assumed that nuclear forces are peripheral, and might be correctly described by meson exchange mechanisms. The minimal set of meson exchange diagrams which should be treated is the sum of all ladder and crossed ladder diagrams. Below the meson production threshold, this set will be regarded as sufficient, but above the production threshold self energies must be included, but only as necessary to insure 3 (and perhaps 4) body unitary. Values of meson and baryon parameters, particularly masses, will be taken as much as possible from known, measured results. The examples developed in this talk are taken from this meson exchange picture, which is regarded as a good description of the long range, peripheral interactions.

At shorter distances, probably inside of a few tenths of a fermi, it is assumed that the physics is best described in terms of the underlying quark and gluon degrees of freedom. It follows that the structure of mesons and baryons, their strong coupling constants and form factors, and measured baryon electromagnetic form factors, are best described in terms of quarks and gluons. Furthermore, these structures are probably non-perturbative in character, requiring either a full treatment of QCD, or some realistic modeling of the non-perturbative (confining) forces in QCD. For the next decade, modeling will probably be necessary and it would be very desirable to develop a relativistic quark cluster model (RQCM) in which both the relativistic nature of the quarks and gluons, and the relativistic motion of the composite mesons and baryons could be treated. One way to begin such a program is introduce Bethe-Salpeter wave functions for the $q\bar{q}$ structure of mesons, and the qqq structure of baryons. Instead of introducing phenomenological confining forces, and calculating these wave functions, we start in the "middle" and parameterize the wave functions themselves, fit the parameters to data, and then use the wave functions to estimate the size of quark structure effects in nuclear systems. This program has been started in collaboration with Warren Buck and Hiroshi Ito of Hampton University, and some initial results are described in Ito's talk included in these proceedings.

The remainder of this talk is divided into three sections. First, in response to the title of this Workshop, some connections will be developed between light front equations and relativistic equations with one particle on-shell. Next, the concepts used in the spectator model will be reviewed, and it will be shown how they are applied to nuclear physics problems. Finally, recent unpublished numerical results will be presented. This work is being done with a number of collaborators, who will be named as the work is described.

Connections

In this section, a connection will be made between the equation with one particle on-shell, and the light front equation. The arguments will be given for the equation describing the bound state of two spin zero particles only, but should be easily generalized.

We start with the Bethe-Salpeter (BS) equation[1] for the bound state vertex:

$$\Gamma(p, P_B) = -i \int \frac{d^4k}{(2\pi)^4} V(p,k)\Delta(k_1)\Delta(k_2)\Gamma(k, P_B) \tag{1}$$

where V is the relativistic kernel (unspecified in this discussion) and $\Delta(k)$ is the propagator for the spin zero nucleon of mass m:

$$\Delta(k) = \frac{1}{m^2 - k^2 - i\epsilon} \tag{2}$$

and k_1 and k_2 are the 4 momenta of particles 1 and 2:

$$\begin{aligned} k_1 &= \frac{1}{2}P_B + k \qquad P_B = k_1 + k_2 = (M_B, \vec{0}) \\ k_2 &= \frac{1}{2}P_B - k \qquad k = \tfrac{1}{2}(k_1 - k_2) = (k_o, \vec{k}) \end{aligned} \tag{3}$$

The disadvantage of the BS equation is that it is 4-dimensional; it requires integration over the relative energy k_o as well as the relative 3-momentum $\vec{k}$. Furthermore, integration over k_o is complicated by the presence of singularities in the Δ's, V, and Γ. The latter problem is usually dealt with by performing a Wick rotation in k_o so that this integration is transformed along the imaginary axis. While this helps for the bound state equation, it does not remove all complications which arrise in the calculation of scattering amplitudes or form factors, and, in any case, the k_o integration still remains.[2]

However, it is possible to "reduce" the BS equation by restricting the k_o integration in some covariant fashion. While some physicists continue to believe that these are "approximations" to the Bethe-Salpeter equation, they are, in fact, new equations which are just as exact (or inexact) as the original BS equation. If the role of these equations is to sum all ladders and crossed ladders, then it can be shown that these new equations not only eliminate this troublesome extra variable, but they also improve the convergence of the series for the kernel V.[3]

A form of the light front Weinberg equation[4] can be obtained in the following hueristic manner. First introduce the light front variables $k_\pm = k_o \pm k_z$ and $k_\perp$ and then note that

$$d^4k = \frac{1}{2}dk_- dk_+ d^2k_\perp = \frac{1}{2}M_b dk_- dx d^2k_\perp \tag{4}$$

where $x = \frac{k_+}{M_B} + \frac{1}{2}$ and varies from $-\infty$ to $+\infty$. Viewed as a complex function of k_-, the propagator has only two simple poles at

$$k_- = \frac{E_\perp^2}{M_B x} - \frac{1}{2}M_B - i\epsilon\ Sgn(x) \tag{5a}$$

$$= -\frac{E_\perp^2}{M_B(1-x)} + \frac{1}{2}M_B + i\epsilon\ Sgn(1-x) \tag{5b}$$

where $E_\perp^2 = m^2 + k_\perp^2$. Note that these two poles lie on opposite sides of the real axis if and only if $0 \le x \le 1$.[5] Hence, if we were to ignore the poles in V and Γ for a moment, we see that the integration over k_- would give zero unless $0 \le x \le 1$, and that closing the k_- contour in the lower half plane, picking up pole (5a) corresponding to putting particle 1 on shell, would give

$$\begin{aligned} -i\int \frac{d^4k}{(2\pi)^4}\Delta(k_1)\Delta(k_2) &= \int_0^1 \frac{dx}{2}\int \frac{d^2k_\perp}{(2\pi)^3}\int dk_\perp M_B \delta(m^2 - k_1^2)\Delta(k_2) \\ &= \int_0^1 \frac{dx}{2}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{1}{E_\perp^2 - M_B^2 x(1-x)} \end{aligned} \tag{6}$$

This procedure could be used to derive a wave equation if we assume the nucleon poles coming from the propagator dominate the physics. The equation which corresponds to this assumption is

$$\Gamma(p_F, P_B) = \int \frac{d^2k_\perp}{(2\pi)^3}\int_0^1 \frac{dx}{2}\ \frac{V(p_F, k_F)\Gamma(k_F, P_B)}{E_\perp^2 - M_B^2 x(1-x)} \tag{7}$$

where k_F is the 4 vector with $k_- = \frac{E_\perp^2}{M_B x} - \frac{1}{2}M_B$. For later use we can solve this for k_o and k_z, obtaining

$$k_F = \left(\tfrac{1}{2}\tfrac{E_\perp^2}{M_B x} - \tfrac{1}{2}M_B(1-x), k_\perp, \tfrac{1}{2}M_B x - \tfrac{1}{2}\tfrac{E_\perp^2}{M_B x}\right) \tag{8a}$$

We note for future reference that if we had used (5b) to evaluate k_-, and defined x as before, the 4 vector would become

$$k_F' = \left(\tfrac{1}{2}M_B x - \tfrac{E_\perp^2}{2M_B(1-x)}, k_\perp, \tfrac{E_\perp^2}{2M_B(1-x)} - \tfrac{1}{2}M_B(1-x)\right) \tag{8b}$$

Equation(7) is precisely the Weinberg equation, if V is chosen properly. It is amusing that we obtained it by restricting particle one to its mass-shell. However, the Weinberg equation is usually assumed to have both particles on the mass-shell, and the propagator is the k_- energy difference derived from τ ordered perturbation theory. This point of view gives the same propagator, but a different V.

The equation with one particle on-shell which I introduced previously[6] can be obtained in a similar heuristic manner. Now we examine the singularities in the complex k_o plane, and in this case there are four:

$$\begin{aligned} k_o &= E_k - \tfrac{1}{2}M_B - i\epsilon \\ &= -(E_k + \tfrac{1}{2}M_B) + i\epsilon \end{aligned} \quad \text{a}$$

$$\begin{aligned} &= \tfrac{1}{2}M_B - E_k + i\epsilon \\ &= E_k + \tfrac{1}{2}M_B - i\epsilon \end{aligned} \quad \text{b} \tag{9}$$

Closing the k_o contour in the lower half plane would now give two terms

$$-i\int \frac{d^4k}{(2\pi)^4}\Delta(k_1)\Delta(k_2) = \int \frac{d^3k}{(2\pi)^3}\int dk_o \left\{\frac{\delta_+(m^2-k_1^2)}{m^2-k_2^2} + \frac{\delta_-(m^2-k_2^2)}{m^2-k_1^2}\right\}$$

$$= \int \frac{d^3k}{(2\pi)^3}\int dk_o \left\{\frac{\delta(E_k - \frac{1}{2}M_B - k_o)}{2E_kM_B(2E_k - M_B)} - \frac{\delta(E_k + \frac{1}{2}M_B - k_o)}{2E_kM_B(2E_k+M_B)}\right\} \tag{10}$$

Note that the first term corresponds to placing particle one on its *positive* energy mass shell, while the second puts particle two on its *negative* energy mass-shell. Since these terms restrict k_o to different values, they cannot be added, and the corresponding integral equation has two channels:

$$\Gamma(p^{\pm},P_B)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{V(p^{\pm},k^{+})\Gamma(k^{+},P_B)}{2E_kM_B(2E_k-M_B)}-\frac{V(p^{\pm},k^{-})\Gamma(k^{-},P_B)}{2E_kM_B(2E_k+M_B)}\right\} \tag{11}$$

where

$$k^{\pm}=(E_k\mp\frac{1}{2}M_B,\vec{k}) \tag{12}$$

and care should be taken not to confuse $k^{\pm}$ with $k_{\pm}$ in Eq.(4); the subscripts refer to light front variables and the superscripts to Eq.(12). Also, do not confuse these $\pm$ channels with the $\pm$ channels arrising in the spin $\frac{1}{2}$ case, which occur in addition to these.

Note that the second channel in Eq.(11) is very small for bound states or scattering energies near $2m$. Only in extreme situations need it be taken into account. One extreme case occurs when $M_B\rightarrow 0$. In this region, each of the propagators in Eq.(11) has a singularity at $M_B\rightarrow 0$, which however is *exactly cancelled* when $M_B=0$.[7] To describe such ultra-relativistic bound states, both channels of Eq.(11) are needed. For applications to the NN system at moderate energies, it is sufficient to use the original equation[6]

$$\Gamma(p^{+},P_B)=\int\frac{d^3k}{(2\pi)^3}\frac{V(p^{+},k^{+})\Gamma(k^{+},P_B)}{2E_kM_B(2E_k-M_B)} \tag{13}$$

Finally, a correspondence between Eq.(11) and Eq.(7) can be drawn. To obtain this, make the following variable transformations[8] in the first and second terms of Eq.(11):

$$\begin{array}{lll} 1^{st}\ term: & k_-=k_o-k_z & \\ & x=\frac{E_k+k_z}{M_B} & 0\le x\le\infty \\ & & \\ 2^{nd}\ term: & k_-=k_o-k_z & \\ & x=\frac{E_k+k_z}{M_B}+1 & 1\le x\le\infty \end{array} \tag{14}$$

This gives the following result

$$\Gamma(p,P_B)=\int\frac{d^2k_\perp}{(2\pi)^3}\left\{\int_0^\infty\frac{dx}{2}\frac{V(p,k_F)\Gamma(k_F,P_B)}{(E_\perp^2-M_B^2x(1-x))}\right.$$
$$\left.-\int_1^\infty\frac{dx}{2}\frac{V(p,k_F')\Gamma(k_F',P_B)}{(E_\perp^2-M_B^2x(1-x))}\right\} \tag{15}$$

where k_F are k'_F are as defined in Eq.(8a) and (8b). We see that (15) is identical to the light front equation we would have obtained if we had kept *all* the poles in Eq.(5) which lie in the *lower* half plane. (The pole (5a) lies in the LHP for $x > 0$, while (5b) does for $x > 1$.) If we were to ignore the arguments of V and Γ, the two terms in Eq.(15) would cancel, restricting the integration in x to $[0, 1]$, as in Eq.(7).

In conclusion, Eq.(7) and (11) differ in two ways: First, there is the variable transformations (14) which are trivial in the sense that they are really no difference at all. Secondly, there is the necessity to collapse the two terms in (15) into a single term. This last difference is significant. It makes the equations really different and is the origin of the lack of manifest rotational invariance of the light front equations.

Concepts and their application

As advertized in the introduction, this section will review the concepts used in the spectator model, and describe their applications. There are three amplitudes which are sufficient for most applications. When bound states are present, the relativistic vertex function with one particle off shell is required. This vertex function is related to a matrix element of the interacting field between the bound state and the spectator particles in the final state, which, for two body bound states, is

$$\Gamma^{(2)}(p, P_B) \simeq S^{(-1)}(p_2) < p_1|\phi(0)|P_B > \tag{16}$$

where $P_B = p_1 + p_2$ and $p = \frac{1}{2}(p_1 - p_2)$ as in Eq.(3) and $S(p_2)$ is the propagator of the off-shell particle 2. Blankenbecker and Cook were the first to introduce this covariant vertex for the deuteron[9], and a complete discussion of its relation to relativistic deuteron wave functions has been given by Remler[10] and Buck and myself[11]. For three body bound states a similar amplitude is needed[12]

$$\Gamma^{(3)}(p, q, P_B) \simeq S^{-1}(p_3) < p_1 p_2|\phi(0)|P_B > \tag{17}$$

and I believe that the idea can be generalized, with some complications, to four or more bodies. For systems with A nucleons, a vertex can be defined in a similar way

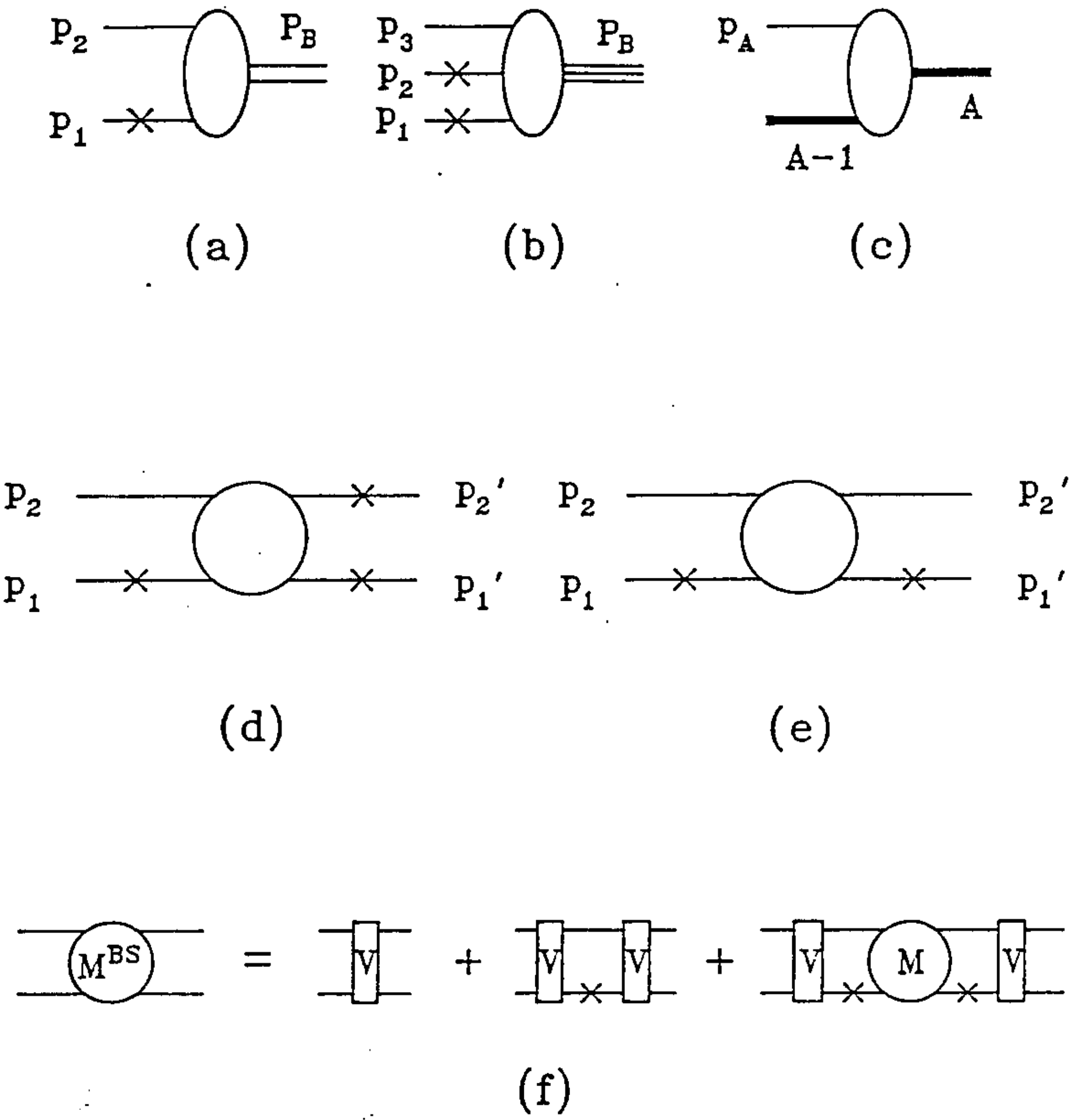


Figure 1: *Concepts in the spectator model. In all diagrams, the cross indicates that the particle is on-shell, and all bound states and A or A-1 particle systems (denoted by a heavy dark line) are also on-shell. Two body bound state vertex functions (a), three body bound state vertex functions (b), vertex function for one off-shell particle in an A body system (c), half off-shell (d) and fully off-shell (e) scattering amplitudes, and equation for the BS amplitude consistent with the spectator model (f).*

$$\Gamma^{(A)}(p_A) \simeq S^{-1}(p_A) < A-1|\phi(0)|A > \tag{18}$$

where nuclear matter can be treated by letting $A \to \infty$. These amplitudes are drawn schematically in Fig. 1a,b, and c.

For scattering problems, the off-shell scattering amplitude is needed. For two body scattering, the half off-shell amplitude is a generalization of (16):

$$M(p,p';P) \simeq S^{-1}\left(\frac{1}{2}\mathrm{P}-\mathrm{p}\right) < \frac{1}{2}\mathrm{P}+\mathrm{p}|\phi(0)|\mathrm{p}',\mathrm{P} > \tag{19}$$

where p' and p are the relative momenta of the two particles in the initial and final state, and the notation is meant to imply that both particles are on-shell in the initial state, but that only particle 1 is on-shell in the final state. Hence

$$\left(\frac{1}{2}P+p\right)^2 = m^2 \tag{20}$$

which becomes a contraint on p_o. In the CM frame, $P = (W,\vec{0})$, this constraint becomes

$$p_o = E_p - \frac{1}{2}W \tag{21}$$

(The same constraint holds for the bound state, of course.) This amplitude is illustrated diagramatically in Fig. 1d. Sometimes the fully off-shell amplitude is required, which is shown diagramatically for the two body system in Fig. 1e. Knowledge of these amplitudes implies knowledge of the relativistic kernel V from which these amplitudes can be calculated by solving the spectator wave equation, which in the CM for two spin zero particles is

$$M(p,p';P) = V(p,p';P) + \int \frac{d^3k}{(2\pi)^3}\frac{V(p,k;P)M(k,p';P)}{2E_kW(2E_k-W)} \tag{22}$$

This is the same kernel which gives the bound state vertex function $\Gamma^{(2)} = \Gamma$ in Eq.(13). In fact, Eq.(13) can be derived from Eq.(22) by using the fact that the existance of the bound state implies a pole in M at $P^2 = P_B^2 = M_B^2$

$$M(p,p';P) = -\frac{\Gamma(p,P)\Gamma^+(p',P)}{M_B^2 - P^2} + R \tag{23}$$

where R is non singular at $P^2 = M_B^2$. Inserting (23) into (22), and demanding that it hold in the vicinity of the pole, gives the bound state equation (13).

It can be shown that the fully off-shell 2 body amplitude shown in Fig. 1e is sufficient to obtain solutions to the relativistic three body Faddeev equations (a three body force term may also be needed), but it appears that a systematic treatment of 4 or more particles may require two body amplitudes with 3 or all 4 particles off shell. Such BS amplitudes can be calculated consistently within the framework of the spectator model provided the kernel V is known for all 4 particles off shell, which will be assumed. The equation, represented diagramatically in Fig. 1f, is

$$M^{BS}(p,p';P) = V(p,p';P) + \int \frac{d^3k}{(2\pi)^3} \frac{V(p,k;P)V(k,p';P)}{2E_k W(2E_k - W)} + \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{V(p,k_1;P)M(k_1,k_2;P)V(k_2,p';P)}{4E_{k_1}E_{k_2}W^2(2E_{k_1} - W)(2E_{k_2} - W)} \tag{24}$$

This definition of M^{BS} is consistent in that $M^{BS} = M$ when one particle is on-shell in both the initial and final state, and the off-shell extrapolation defined in (24) is precisely the amplitude which arises in cases where the spectator model does not uniquely define spectators in either the initial or final state. I will not persue the many (greater than 3) body problem further in this talk as the ideas are just being developed and in a state of flux.

The final concepts needed are the vertex functions which describe how probes interact with nucleons or mesons. For nucleon scattering, the probe is the nucleon itself, and the amplitudes needed are the M matrices just discussed. For pion scattering, the πNN vertex function, and the πN scattering amplitudes are needed. The πNN vertex function is contained in V, and work is underway to apply the spectator model needed to πN scattering; this will not be discussed further. Finally, for electron scattering, the off shell γNN, $\gamma\pi\pi$, and other current "operators" are needed. Recently, a way has been found to introduce the currents in such a way that gauge invariance is satisfied exactly, and pheonomological form factors (both electromagnetic and strong, such as those used at the πNN vertex) may be used without constraints.[13] This is achieved by reinterpreting the strong form factors as self energies, and modeling the off shell current so that it is consistent with the Ward-Takahashi identities which result:

$$q_\mu j^\mu(k',k) = \Delta^{-1}(k') - \Delta^{-1}(k) \tag{25}$$

This development removes the last obstacle to using the spectator approach to calculate electromagnetic processes consistently, and V. Dmitrasinovic (William and Mary), J.W. VanOrden(CEBAF) and I plan to use the NN models discussed below to calculate electromagnetic interactions involving deuterons.

Figure 2 illustrates how the concepts discussed above are used in applications. Figs. 2a and b show the relativistic impulse approximation to the deuteron form factor and the three-body form factor (in the three body case, the $pd\ ^3He$ vertex is also required). The relativistic bound state vertex functions and the off-shell nucleon form factors are required, and the spectators to the electromagnetic interactions are on-shell. I originally viewed these diagrams as an approximation to the full diagrams with all internal particles off-shell[14], but I now believe that these should be viewed as one term (probably the largest) in the exact current operator, the structure of which is largely determined by the dynamical content of the two body interaction kernel V.[13] (The other terms can be determined from V; in a OBE model for V, there are only 3 kinds of terms, all of which can be calculated.)

Figures 2c, d, and e show various contributions to electrodisintegration. Fig. 2e is particularly amusing; this impulse diagram requires precisely the bound state amplitude calculated in the spectator model! Figure 2d is a meson exchange contribution (MEC), and 2e is the final state interaction (FSI). In Fig. 2e, the spectator is again on shell, and the half off-shell amplitude calculated in Eq. (22) is just what is required for the rescattering. Note that in the MEC Fig. 2d, there is no unique spectator, and two diagrams, one with particle one on-shell and one with particle two on-shell, are required; only one of these is shown in Fig. 2d.

Figure 2f shows how spectators can be uniquely identified if three body scattering is regarded as a succession of two body scatterings. Using this analyses, relativistic three body Faddeev equations, driven by the off-shell amplitude shown in Fig. 1e, can be derived. In Fig. 2g the self consistent equations for an A body bound state are written down diagramatically. The kernel used in this equation is identical to the optical potential [15] required for relativistic proton nuclear scattering, and results using such a potential will be reported in the next section. In the limit when $A \to \infty$, if the two body scattering amplitude M is approximate by its born term V, it can be shown that only σ and ω exchanges survive, and the mean field results of Serot and Walecka can be obtained.[16] Hence the concepts of the spectator model can be applied to few and many body problems in a consistent fashion.

The discussion presented in this section has been very heuristic, but it is possible to develop the discussion in a more formal and rigorous manner.

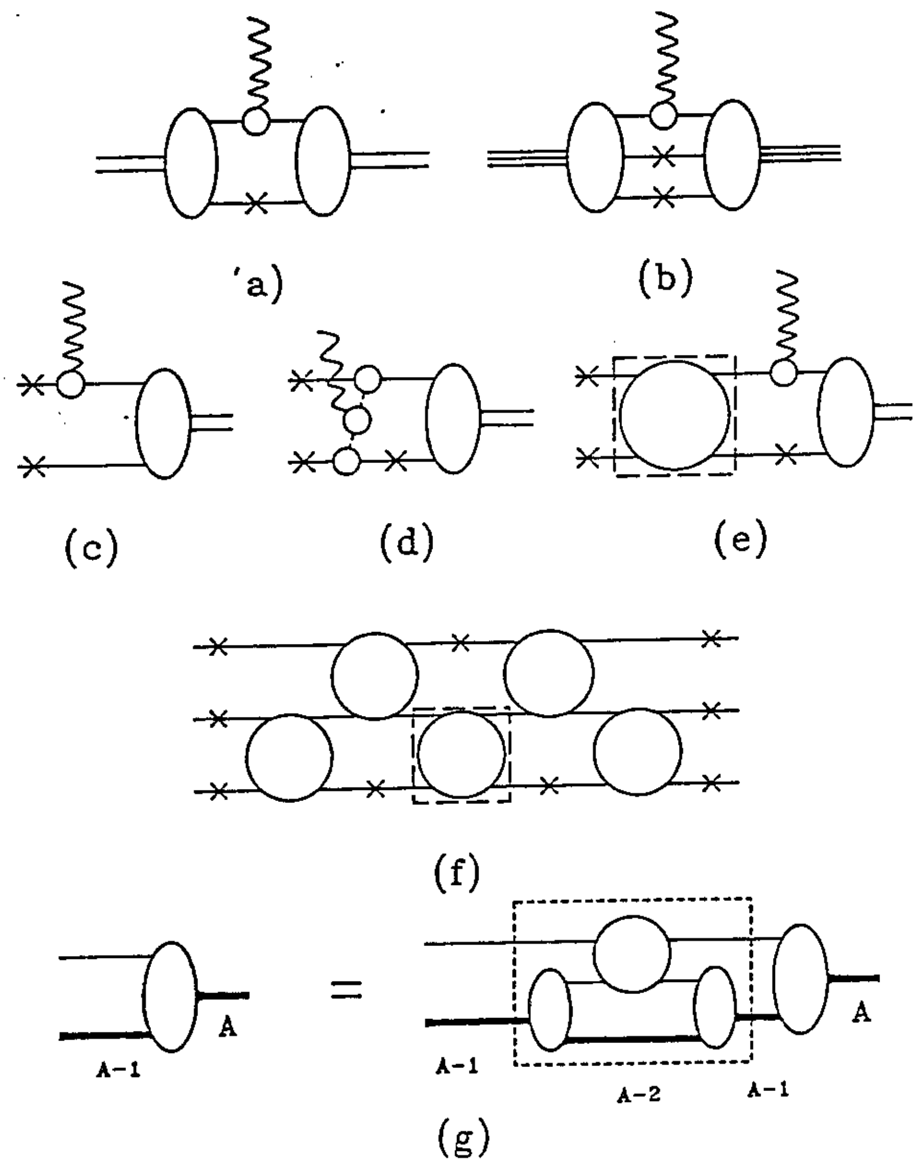


Figure 2: *Applications of concepts in the spectator model. Symbols are described in figures or the text. Two (a) and three (b) body form factors in the impulse approximation; d(e, e′ p)n diagram for the RIA(c), MEC(d), and FSI(e) with the half off-shell amplitude shown in the dashed box; typical sequence of two particle scattering which drives the three body amplitudes are shown in (f) with fully off-shell two body amplitude shown in the dashed box; equation for A nucleon bound state shown in (g) with potential for p-nucleus scattering shown in the dashed box.*

The advantages of the spectator model are

(i) it is manifestly covariant; the properties of all amplitudes under the Lorentz group can be written down explicitly, and all amplitudes conserve energy and momentum, as required by space-time translational invariance;

(ii) there is a close connection to field theory through its expansion in Feynman diagrams, permiting the dynamics of meson exchange to be introduced in a natural way;

(iii) the non-relativistic limits of all amplitudes can be obtained naturally in the $m \to \infty$ limit, establishing a close correspondence with non-relativistic theory and facilitating interpretation of all quantities;

(iv) it can be shown [3] that the kernel V is rapidly convergent in the $m \to \infty$ limit, providing a smooth transition from two body equations to one body equations; and

(v) there is cluster separability; for example, the 3 body equations are driven by the same two body amplitudes calculated in the two body problem.[12]

There are two disadvantages of the spectator approach, only one of which is serious, in my opinion. The non-serious disadvantage is that the equations appear to be unsymmetric because only one of the two particles is on-shell. When dealing with identical particles, where symmetry is required, it can be obtained by explicitly symmetrizing the kernel, as illustrated in Fig. 3a for the OBE model. Once this is done, it can be shown that the two body

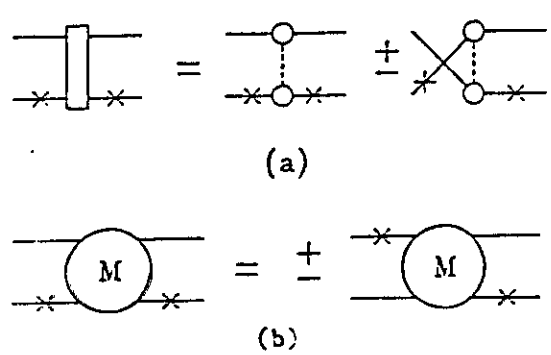


Figure 3: *(a)Symmetrization of the relativistic kernel, and (b)resulting symmetry of the M matrix.*

amplitude is fully symmetric, as shown in Fig. 3b. Alternatively, with a symmetrized kernel it can be shown that an equivalent form of the equation can be writtein in which the propagator is an equal mixture of terms with particle one on-shell and terms with particle two on-shell. Hence, while the equations may look unsymmetric, they are in fact fully symmetric for identical particles, and the Pauli principle for two identical spin $\frac{1}{2}$ nucleons is satisfied exactly.

A second disadvantage is more serious: the process of putting particles on-shell introduces spurious singularities into the interaction kernels. (These are not singularities associated with particle production, which are expected, but singularities which have no physical origin.) The singularity in the propagator as $M_B \to 0$, which was discussed in the previous section, is an example of such a singularity; it arose because a "distant" negative energy pole was ignored which is not "distant" and can not be ignored when $M_B \to 0$. The spurious singularities which occur in the kernel have a similar origin. It can be shown[3] that such singularities arise from the way in which Feynman diagrams are divided into spectator and non-spectator pieces, and that when all pieces are added together, these singularities cancel. This cancellation may therefore be used to justify dropping the imaginary parts of these singularities. It does not appear that the real parts (principal values) of the singularities can be discarded without greatly adding to the complexity of the equations, but as they occur only when at least one particle is off-shell, and therefore appear only in virtual intermediate states (which are integrated over), and as they occur only at rather large momenta, they seem to have a negligible effect on the numerical results and can be accepted as one of the features of this phenomenology. Their numerical influence is presently being studied in detail in collaboration with J.W. Van Orden.

Recent numerical results

This section will report on recent fits to the NN scattering phase shifts and their first application: the predictions for $\bar{p}\,{}^{40}Ca$ scattering observables.

Work on using Eq.(13) (suitably generalized to describe two spin $\frac{1}{2}$ particles) to describe the deuteron and NN scattering phase shifts has been underway for some time. The non-relativistic limit of this equation was studied some time ago[17], and numerical solutions for the deuteron in an OBE model have also been obtained.[11] The present work began in collaboration with K. Holinde (Julich) who brought an early version of the Bonn phase shift code to Williamsburg. Recently, J.W. VanOrden has made substantial improvements in the code, and we now can automatically vary the OBE parameters to obtain a best fit to the phase shifts, scattering lengths, effective ranges, and deuteron binding energy. This work is still in progress, and there will be small changes in the results I will report on here, but the essential features are clear at this time.

The OBE models presented here have the following features:

(i) The coupling of pseudoscalar mesons (π and η) includes an off-shell mixing parameter λ_m

$$g_m \left[\lambda_m \gamma^5 + (1-\lambda_m) \frac{(\not p_f - \not p_i)}{2m} \gamma^5 \right] \tag{26}$$

(ii) The coupling of vector mesons (ρ and ω) also includes an off-shell mixing parameter λ_m

$$g_m \left\{ 1 + \kappa_m(1-\lambda_m) + \lambda_m \frac{\kappa_m}{2m} i\sigma^{\mu\nu}(p_f - p_i)_\nu \right. \\ \left. -(1-\lambda_m)\kappa_m \frac{(p_f+p_i)^\mu}{2m} \right\} \tag{27}$$

defined in such a way that the coupling is independent of λ_m when both the initial and final nucleon are on-shell. Note that this mixing parameter gives off shell sensitivity only when the tensor coupling κ_m is non-zero. Since the tensor coupling of the ω meson is small, λ_ω was fixed at unity.

(iii) All meson nucleon vertices have the same phenomenological form factor

$$F(q^2) = \left(\frac{\Lambda^2 - \mu_m^2}{\Lambda^2 - q^2} \right)^3 \tag{28}$$

where Λ is an adjustable parameter (the same for all mesons), μ_m is the meson mass, and q^2 is the square of the 4-momentum carried by the meson.

(iv) The off-shell nucleon carries a form factor of the form

$$F_N(p^2) = \left(\frac{\Lambda_N^2 - m^2}{\Lambda_N^2 - p^2} \right)^4 \tag{29}$$

This form factor is essential for convergence of the equations.

(v) Both the direct and exchange terms shown in Fig. 3a use the form of the four vector q^2 appropriate to the direct term

$$q^2 = (p_f - p_i)^2 = (E_f - E_i)^2 - (\vec{p}_f - \vec{p}_i)^2 \tag{30}$$

New fits currently being prepared will relax the restriction given in (v), and use the q^2 appropriate to each diagram. This will also require new form factors. Some fits of this kind have already been obtained, and the results do not differ significantly from those to be presented here.

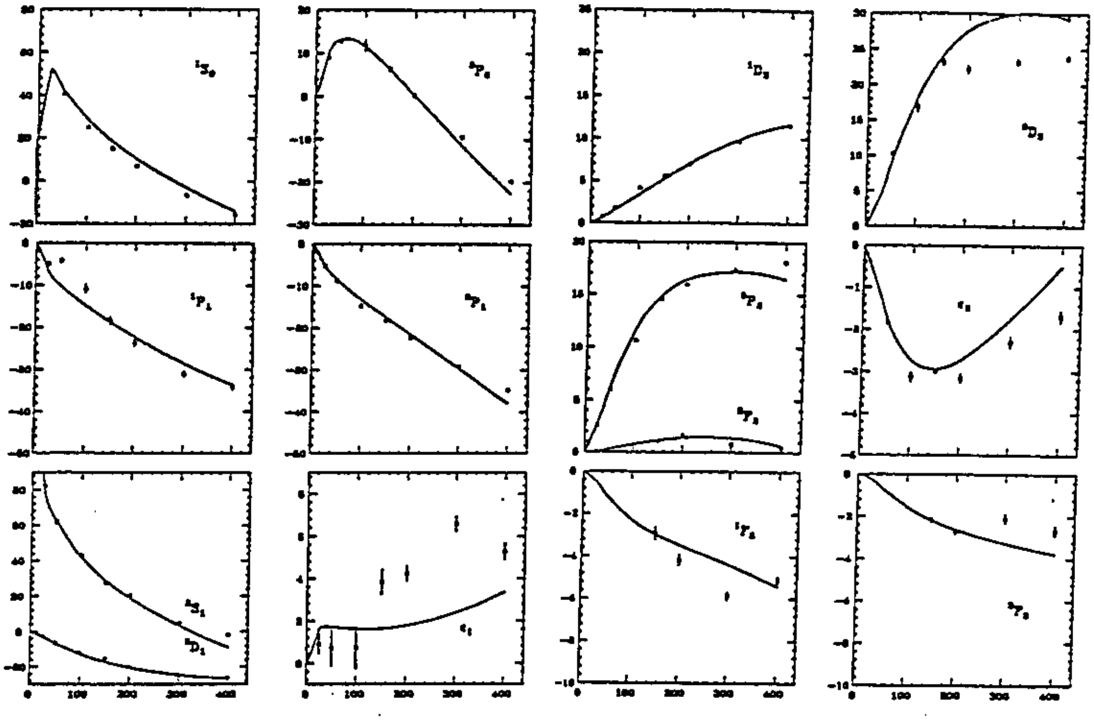
Two OBE models have been found which fit the NN observables very well. The fits to NN phase shifts below 400 MeV are shown in Figures 4 and 5. While there are some differences between the fits, these differences are small, and it is not misleading to regard both models as fitting the phase shifts equally well. Yet the two models differ significantly in their dynamical content. Model 1 includes the exchange of only the 4 mesons essential to any OBE description of nuclear forces: π, σ, ρ, and ω. The mixing parameter $\lambda_\pi = 0.25$, corresponds to a 25% admixture of γ^5 coupling for the pion. Model 2 constrains $\lambda_\pi = 0$, giving a pure $\gamma^5\gamma^\mu$ coupling for the pion which many physicists believe is required by pair suppression and chiral symmetry.

meson	$\frac{g_m^2}{4\pi}$	κ_m	λ_m	μ_m
π	13.83		0.25	
	14.11		0.00*	
σ	4.26			491
	4.65			510
ρ	0.40	7.22	0.97	
	0.65	6.20	0.77	
ω	7.49	0.26		
	8.79	0.02		
η	–		–	
	5.26		0.47	
δ	–			–
	0.41			520

$\Lambda =$ 2760 $\Lambda_N =$ 1930
2250 2000

Table 1

*Constrained in Model 2.



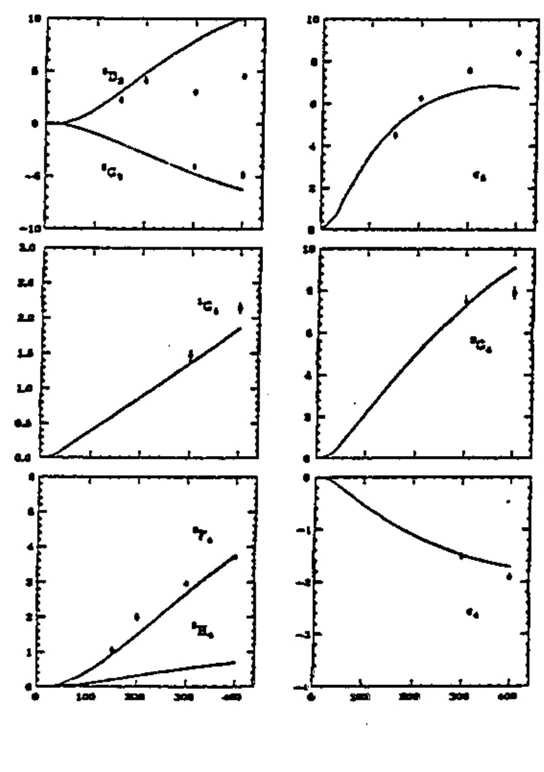


Figure 4: *Phase shift fit for Model 1*

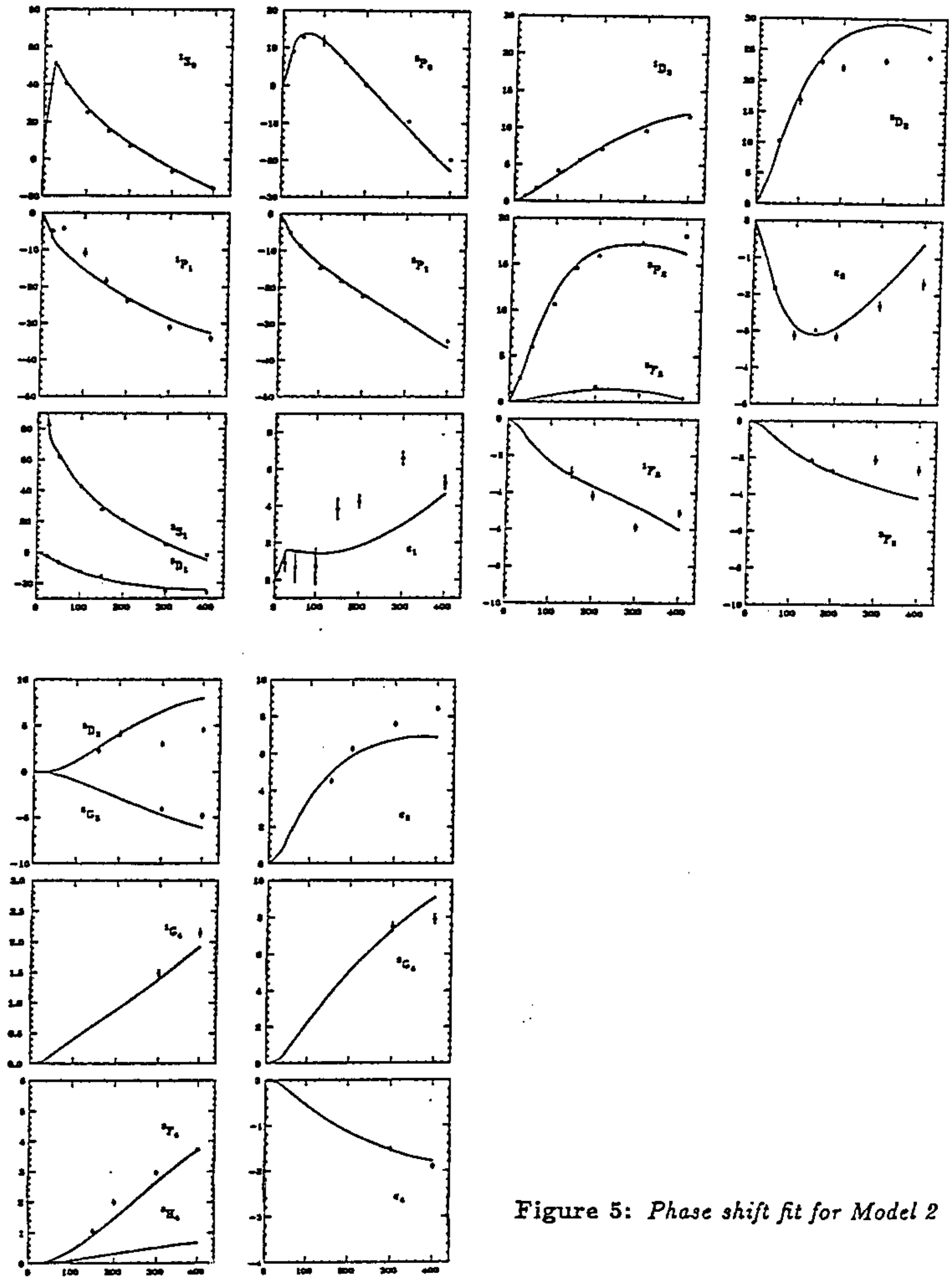


Figure 5: *Phase shift fit for Model 2*

To obtain an equally good fit to the phase shifts, this model requires an additional η and δ meson. The δ meson is needed to give splitting between ^1S_o and $^3S_1$ phase shifts, which arises naturally when $\lambda_\pi \neq 0$. The parameters of the two models which were adjusted by the fit are shown in Table 1. The top number in the table is the value of the parameter for Model 1; the second number for Model 2. Note that the values of the parameters are reasonable for both models, to the extent that they are known, and that the omega meson coupling constant is smaller than that required by most other models. This reflects the fact that some of the short range repulsion comes from contributions of negative energy states, requiring the omega to provide less of the needed repulsion.[17]

Deuteron wave functions have been calculated for both models. They differ significantly in the size of their relativistic components. The largest of these is always the triplet P state, which is 0.51% in Model 1, but only 0.12% in Model 2.

The first real test of Models 1 and 2 has been done in collaboration with Khin Maung Maung (Old Dominion University and NASA), John Tjon (Utrecht), Larry Townsend (NASA), and S.J. Wallace (UMd).[18] The relativistic impulse approximation for the proton nucleus optical potential, shown in Fig. 2g, was evaluated for a ^{40}Ca target using (i) relativistic ^{40}Ca densities supplied by B. Serot and C. Horowitz[19], (ii) the NN scattering matrices determined by Models 1 and 2, and (iii) the Dirac scattering code of Tjon and Wallace.[15] No parameters are adjusted in this calculation; the results, which are an absolute prediction, are shown in Fig. 6. Note that both models give a qualitatively good description of the $\vec{p}\ ^{40}Ca$ observables at 200 MeV, and results at other energies (up to 500 MeV) are just as good. It is amusing that Model 1, with only four mesons and 25% γ^5 coupling for the pion, fits these observables as well as it does. We look forward to the prediction of these two (and other) models for the deuteron electro- and photo-disintegration observables.

Conclusion

The spectator model is being vigorously persued. Preliminary results for NN observables below 400 MeV have been obtained which give a very good description of the phase shifts and deuteron binding energy. These same models also describe $\vec{p}\ ^{40}Ca$ observables quite well. A new generation of calculations of two body electromagnetic observables (deuteron form factors,

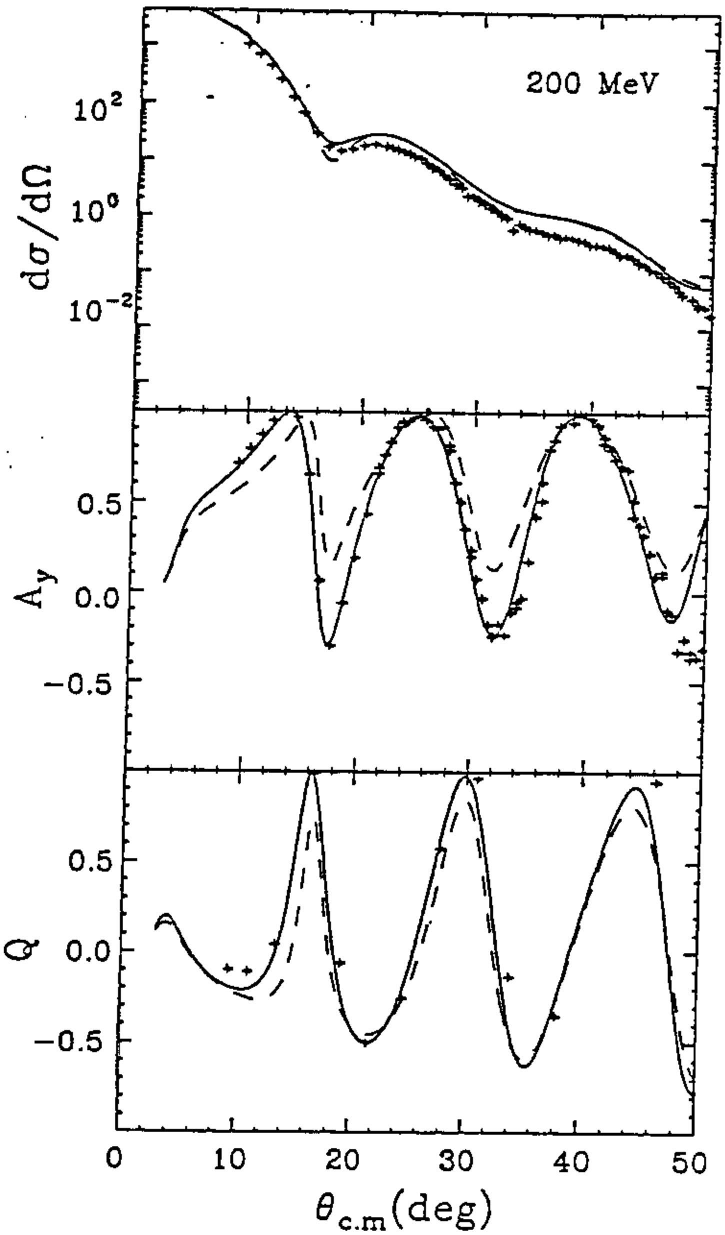


Figure 6: *Prediction for the $\bar{p}$ ^{40}Ca scattering observables at 200 MeV. Solid lines are Model 1, dashed lines are Model 2. Crosses are data points.*

threshold electrodisingetration, and $d(e,e'p)n$ measurements) is underway. These calculations will use a relativistic current operator consistent with the relativistic OBE kernels used in the NN interaction, and will be exactly gauge invariant. Calculations of nuclear matter and three body bound states, consistent with the spectator model and the two body dynamics, are planned for the future.

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